Modeling decomposable Mixed Integer Programs

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Plan

- **BlockDecomposition.jl - Modeling**
- **BlockDecomposition.jl - Pricing Callbacks**
- **RCSP.jl - Pricing Callback Generator**
- **Supported solvers**

In following slides

```
const BD = BlockDecomposition
const RM = RCSP.Modeling
const RS = RCSP.Solver
```

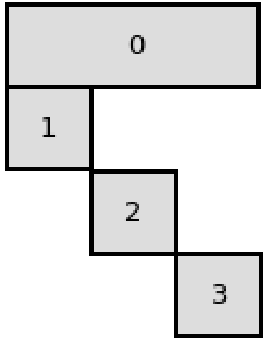
BlockDecomposition.jl

Modeling

Decomp.

Dantzig-Wolfe

We partition constraints.



- Constraints mc_1 to mc_m are in the master.
- Constraints $sc_{1,1}$ to $sc_{1,o}$ are in the 1st subproblem.
- Constraints $sc_{2,1}$ to $sc_{2,p}$ are in the 2nd subproblem.
- Constraints $sc_{3,1}$ to $sc_{3,q}$ are in the 3rd subproblem.

A function to describe this decomposition

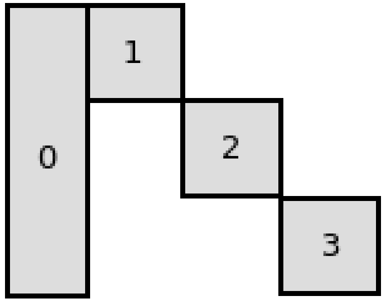
```
function dw_decomp(constr_name, constr_id)
    if constr_name == :mc
        return (:DW_MASTER, 0)
    else
        return (:DW_SP, constr_id[1])
    end
end
BD.add_dantzig_wolfe_decomposition(m, dw_decomp)
```

Decomp.

Dantzig-Wolfe

Benders

We partition variables.



- Variables y_α, $\alpha \in 1 \ldots h$ are in the master.
- Variables $x_{1,\alpha}$, $\alpha \in 1 \ldots i$ are in the 1st subproblem.
- Variables $x_{2,\alpha}$, $\alpha \in 1 \ldots j$ are in the 2nd subproblem.
- Variables $x_{3,\alpha}$, $\alpha \in 1 \ldots k$ are in the 3rd subproblem.

A function to describe this decomposition

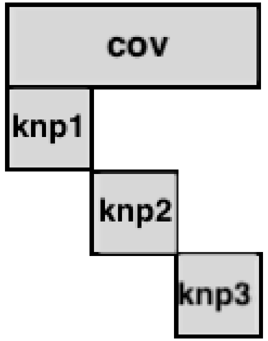
```
function b_decomp(var_name, var_id)
    if var_name == :y
        return (:B_MASTER, 0)
    else
        return (:B_SP, var_id[1])
    end
end
BD.add_benders_decomposition(m, b_decomp)
```

Decomp.

Dantzig-Wolfe

Benders

Example



Generalized Assignment Problem

Assign each job to a machine at minimum cost while not exceeding capacities of machines.

Let x_{mj} equals 1 if job j is assigned to machine m; 0 otherwise.

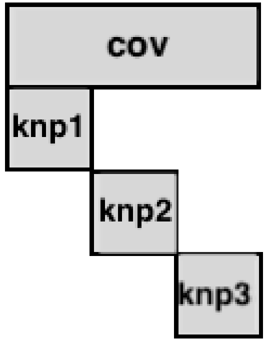
```
gap = Model(solver = Solver())

@variable(gap, x[m in Machines, j in Jobs], Bin)

@constraint(gap, cov[j in Jobs],
               sum( x[m,j], m in Machines ) >= 1)

@constraint(gap, knp[m in Machines],
               sum(weight[m,j]*x[m,j], j in Jobs) <= capacity[m])

@objective(gap, Min,
               sum(cost[m,j]*x[m,j], m in Machines, j in Jobs))

solve(gap)
```

Decomp.

Dantzig-Wolfe

Benders

Example




```
gap = BD.BlockModel(solver = BaPCodSolver())

@variable(gap, x[m in Machines, j in Jobs], Bin)

@constraint(gap, cov[j in Jobs],
               sum(x[m,j], m in Machines) >= 1)

@constraint(gap, knp[m in Machines],
               sum(weight[m,j]*x[m,j], j in Jobs) <= capacity[m])

@objective(gap, Min,
               sum(cost[m,j]*x[m,j], m in Machines, j in Jobs))

function dw_fct(cstr_name, cstr_id)
    if cstr_name == :cov            # cov constraints are assigned to
        return (:DW_MASTER, 1)      # the master that has the index 1
    else                            # knp constraints are assigned to
        return (:DW_SP, cstr_id)    # the subproblem with the same id
    end
end

BD.add_dantzig_wolfe_decomposition(gap, dw_fct)
```

BlockDecomposition.jl

Pricing Callbacks

Pricing callback

Definition

Pricing callbacks can be used to solve efficiently subproblems.

Available functions :

```
function BD.getcurcost(cb, var)::Float64
function BD.getcurub(cb, var)::Float64
function BD.getcurlb(cb, var)::Float64
function BD.setsolutionvalue(cb, var, value)::Void
```

We introduce them with the Generalized Assignment Problem.

Pricing callback

Definition

Example

A function solving efficiently the knapsack problem.

```
(sol,value) = solveKnp(costs, weights, capacity)
```

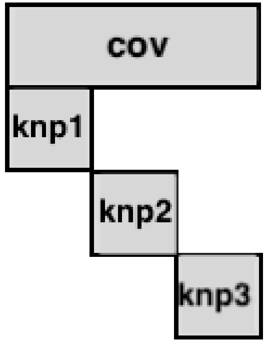
A pricing callback using this function:

```
function myKnapsackSolver(cb)
    machine = BD.getspid(cb)[1] # machine index

    costs = [BD.getcurcost(x[machine,j]) for j in Jobs]

    (sol_x_m, value) = solveKnp(costs, Weight[m,:], Capacity[m])

    for j in data.jobs
        BD.setsolutionvalue(cb, x[machine,j], sol_x_m[j])
    end
end
```

Pricing callback

Definition

Example

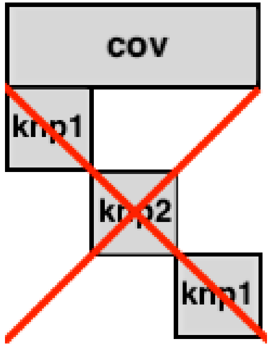



```
gap = BD.BlockModel(solver = BaPCodSolver())

@variable(gap, x[m in Machines, j in Jobs], Bin)

@constraint(gap, cov[j in Jobs],
               sum(x[m,j], m in Machines) >= 1)
@constraint(gap, knp[m in Machines],
               sum(weight[m,j]*x[m,j], j in Jobs) <= capacity[m])
@objective(gap, Min,
               sum(cost[m,j]*x[m,j], m in Machines, j in Jobs))

function dw_fct(cstr_name, cstr_id)
    if cstr_name == :cov
        return (:DW_MASTER, 1)
    else
        return (:DW_SP, cstr_id)
    end
end
BD.add_dantzig_wolfe_decomposition(gap, dw_fct)

# Pricing callback assignment
for m in Machines
    BD.addpricingcbtosp!(gap, m, myKnapsackSolver)
end
```

Pricing callback

Definition

Example




```
gap = BD.BlockModel(solver = BaPCodSolver())

@variable(gap, x[m in Machines, j in Jobs], Bin)

@constraint(gap, cov[j in Jobs],
               sum(x[m,j], m in Machines) >= 1)

@objective(gap, Min,
               sum(cost[m,j]*x[m,j], m in Machines, j in Jobs))

# Decomposition on constraints
dw_fct(cstr_name, cstr_id) = (:DW_MASTER, 1)
BD.add_dantzig_wolfe_decomposition(gap, dw_fct)

# Decomposition on variables
dw_fct_on_vars(var_name, var_id) = (:DW_SP, var_id[1])
BD.add_dantzig_wolfe_decomposition_on_variables(gap, dw_fct_on_vars)

# Pricing callback assignment
for m in Machines
    BD.addpricingcbtosp!(gap, m, myKnapsackSolver)
end
```

RCSP.jl

Resource Constrained Shortest Path Pricing Callback Generator

RCSP

Definition

- Structure of the network
- Variables to edges assignment
- Edges / Vertices resources properties
  - consumption and bounds

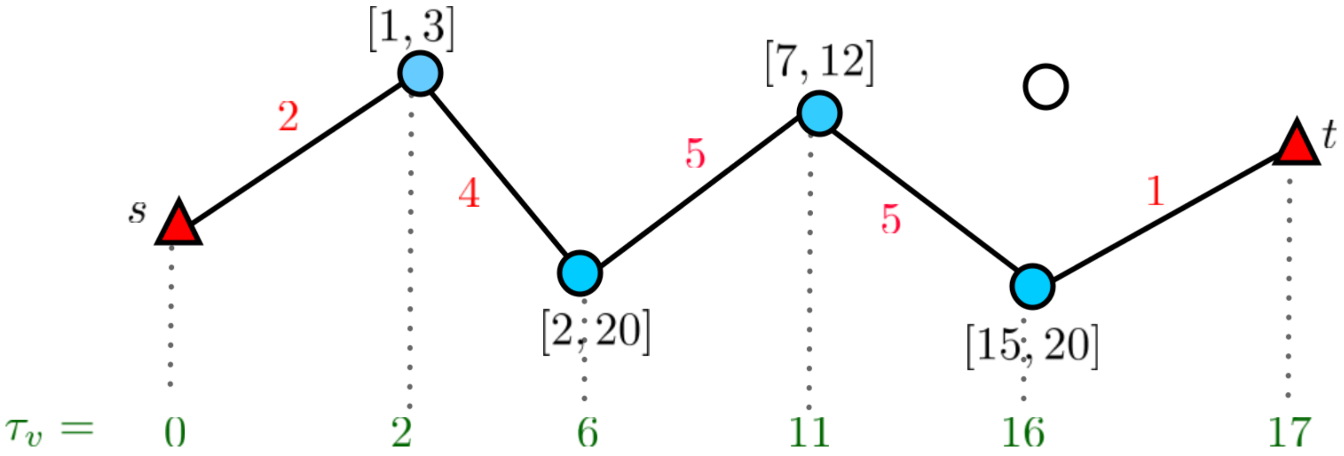


Figure: Example of RCSP feasible solution

Heterogeneous Vehicle Routing Problem With Time Windows (HVRPTW) formulation :

$$
\begin{aligned}
\min \quad & \sum_{k=1}^{U} \sum_{i,j} c_{ij}^k x_{ij}^k \\
\text{s.t.} \quad & \sum_{k=1}^{U} \sum_{i,j} x_{ij}^k = 2 && j \in V \setminus \{depot\} \\
& x^k \in X^k && k = 1..U
\end{aligned}
$$

- $x_{ij}^k = 1$ if edge (i, j) is used by vehicle k
- c_{ij}^k cost of edge (i, j) for vehicle k
- U number of heterogeneous vehicles
- X^k set of tours visiting a subset of customers within their time windows that vehicle k can do.

Tours X^k are generated for each vehicle k by a pricing callback.

RCSP

Definition

Example

Formulation

Compact formulation + decomposition functions.

```
vrp = BD.BlockModel(solver = BaPCodSolver())

@variable(vrp, x[k in K, a in Arcs], Int)

@constraint(vrp, part[c in C],
                 sum(x[k, a] for k in K, a in incident_arcs(c)) == 2.0)

@objective(vrp, Min, sum(cost(k, a) * x[k, a] for k in K, a in Arcs))

# Decomposition on constraints
dw(cstr_name, cstr_id) = (:DW_MASTER, 0)
BD.add_dantzig_wolfe_decomposition(vrp, dw)

# Decomposition on variables
dw_on_vars(var_name, var_id) = (:DW_SP, var_id[1])
BD.add_dantzig_wolfe_decomposition_on_variables(vrp, dw_on_vars)
```

RCSP

Definition

Example

Formulation

RCSP cb.

For a given vehicle k, tours are generated solving a RCSP.

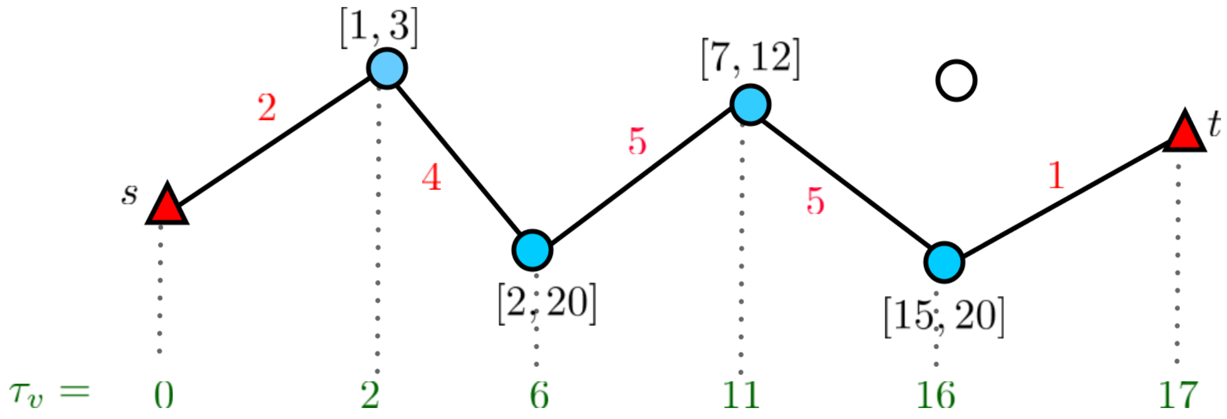


Figure: Example of feasible solution

- Variable x_{ij}^k is assigned to edge (i, j)
- Resource is time
- Resource consumption on edge is **travel time** of vehicle k.
- Bounds on accumulated resource consumption at vertices are **time windows.**

Network is the road network.

```
network = RM.Network(nb_nodes, source = 1, sink = nb_customers + 2)
```

Resource is time.

```
time_res = RM.addresource!(network)
```

Definition of time windows.

```
for v in Vertices
  RM.setresourceproperties!(network, v, time_res, lb = a(v), ub = b(v))
end
```

Instantiation of edges.

```
for c1 in Customers, c2 in Customers
  if c1 != c2
    edge = RM.add_edge!(network, (c1, c2), var = x[k, (c1, c2)])
    RM.setresourceproperties!(network, edge, time_res,
        consumption = traveltime(k, c1, c2))
  end
end

for c in Customers
  # Source
  edge = RM.add_edge!(network, depot, c, var = x[k, (depot, c)])
  RM.setresourceproperties!(network, edge, time_res,
      consumption = traveltime(k, depot, c))

  # Sink
  edge = RM.add_edge!(network, c, sink, var = x[k, (depot, c)])
  RM.setresourceproperties!(network, edge, time_res,
      consumption = traveltime(k, c, sink))
end
```

A function wrapping the definition of the RCSP problem.

```
function vrptw_rcsp(cb)
  k = BD.getspid(cb)[1] # Get the vehicle id
  network = RM.Network(nb_customers + 2)

  # Define the network, resource, etc .

  return network
end
```

Generation of a pricing callback for each subproblem.

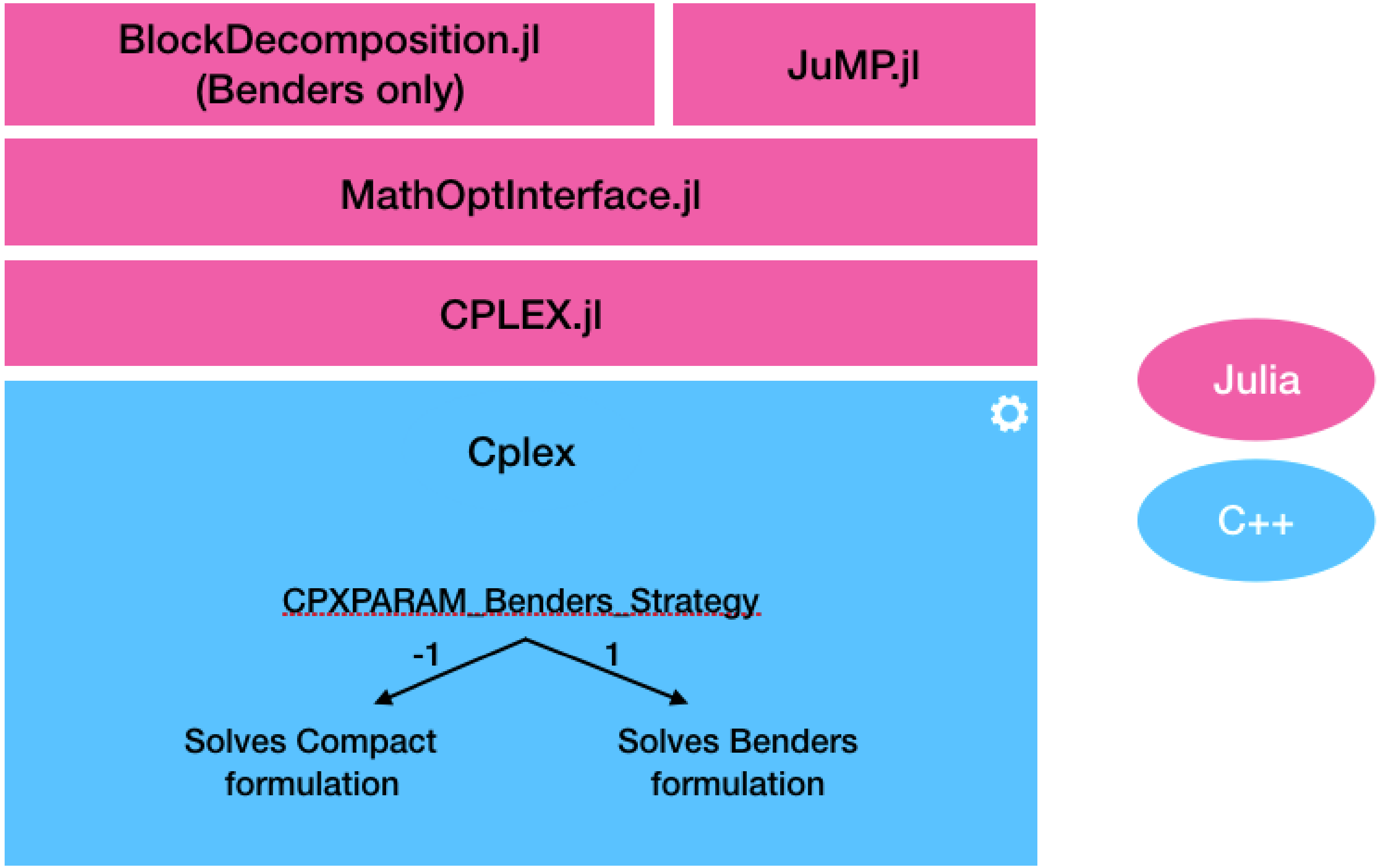
```
for k in K
  RS.generate_rcsp_callback!(vrp, (:DW_SP, k), vrptw_rcsp)
end
```

Multiplicity equals the number of vehicles of each type.

```
BD.addspmultiplicity(vrp, (spid, sptype) -> (0, 1))
```

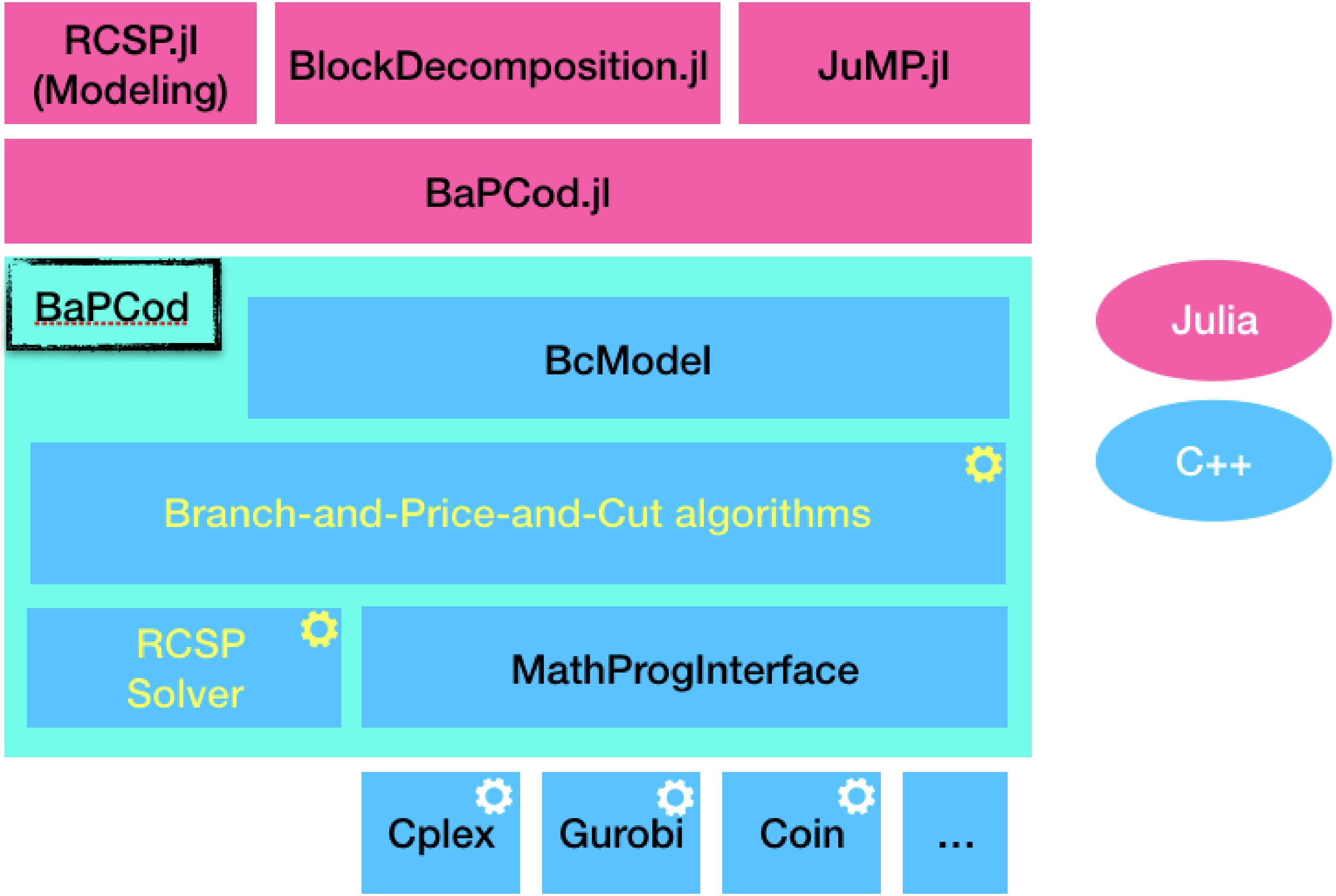
Supported solvers

CPLEX

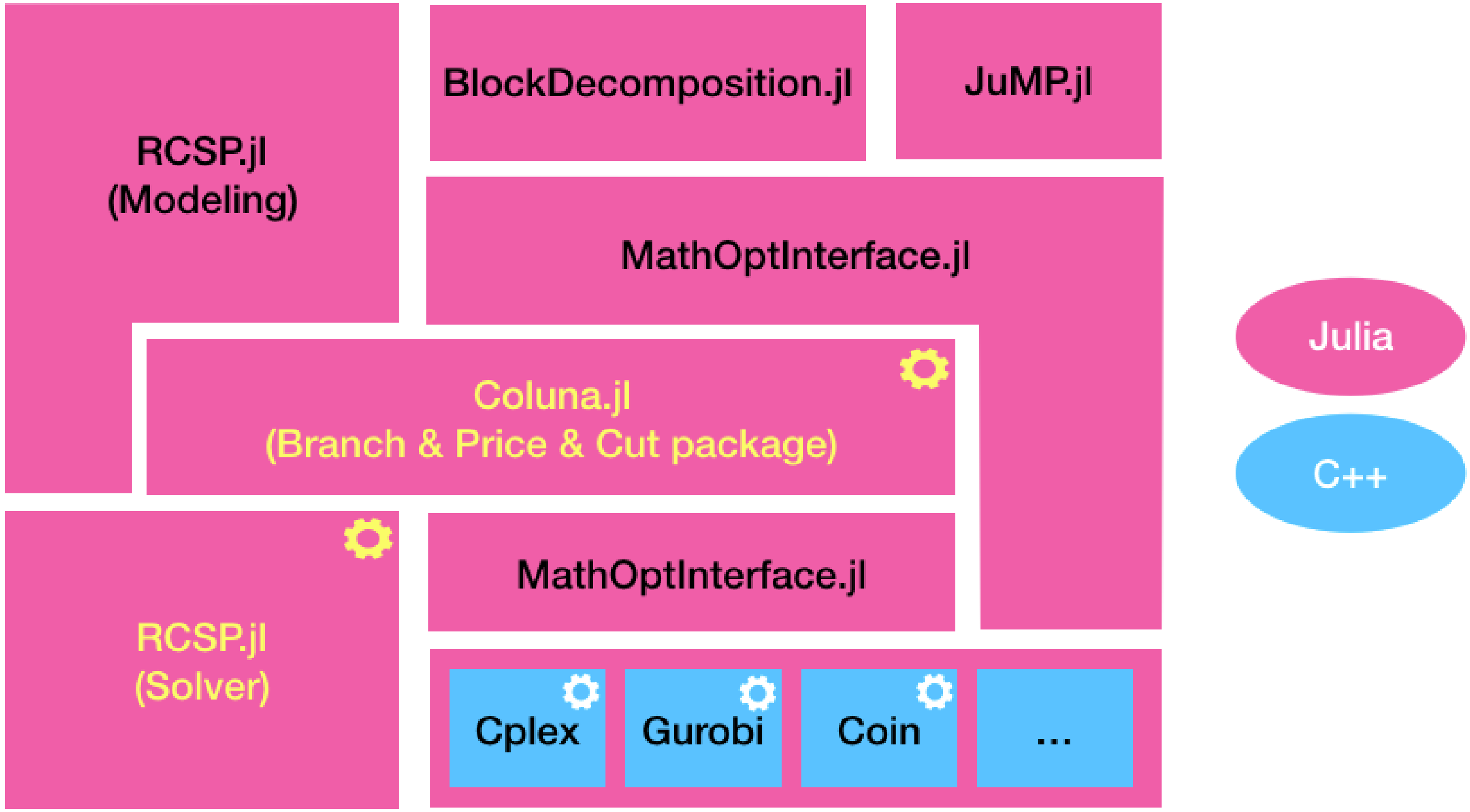


BlockDecomposition.jl
(Benders only)
JuMP.jl
MathOptInterface.jl
CPLEX.jl
Cplex
CPXPARAM_Benders_Strategy
-1
1
Solves Compact formulation
Solves Benders formulation
Julia
C++

BaPCod


RCSP.jl
(Modeling)
BlockDecomposition.jl
JuMP.jl
BaPCod.jl
BaPCod
BcModel
Branch-and-Price-and-Cut algorithms
RCSP
Solver
MathProgInterface
Cplex
Gurobi
Coin
...
Julia
C++

Coluna.jl (ongoing work)



Thank you!

Questions?